



## Maximal Parabolic Regularity Under Time Discretization

**Christian Lubich,**

Professor, University of Tübingen, Germany



### Περίληψη – Abstract

Maximal regularity is an important mathematical tool in studying existence, uniqueness and regularity of the solution of nonlinear parabolic partial differential equations. In this talk the following question is addressed:

Given an operator on a Banach space that has maximal  $L^p$ -regularity (for  $1 < p < \infty$ ), for which (if any) time discretization methods for the associated parabolic initial value problem is the maximal regularity preserved in the discrete  $\ell^p$ -setting, uniformly in the stepsize?

It is found that the time discretization by a linear multistep method or Runge–Kutta method has maximal  $\ell^p$ -regularity uniformly in the stepsize if the method is A-stable (and satisfies minor additional conditions). The notion of A-stability was introduced by Dahlquist in 1963 and is a well-known concept in the study of time discretization methods. In particular, the implicit Euler method, the Crank–Nicolson method, the second-order backward difference formula (BDF), and the Radau IIA and Gauss Runge–Kutta methods of all orders preserve maximal regularity. The proof uses Weis' characterization of maximal  $L^p$ -regularity in terms of R-boundedness of the resolvent, a discrete operator-valued Fourier multiplier theorem by Blunck, and generating function techniques that have been familiar in the stability analysis of time discretization methods since the work of Dahlquist. The  $A(\alpha)$ -stable higher-order BDF methods have maximal  $\ell^p$ -regularity under an R-boundedness condition in a larger sector.

As an illustration of the use of maximal regularity in the error analysis of discretized nonlinear parabolic equations, it is shown how error bounds are obtained without using any growth condition on the nonlinearity or for nonlinearities having singularities.

**Christian Lubich** was born in 1959. He obtained his Ph.D. and his Habilitation from the University of Innsbruck in 1983 and in 1987, respectively. After having held Assistant and Visiting Professor positions at the Universities of Innsbruck, Heidelberg, Geneva, Rennes, and at the ETH Zürich, he became a Professor of Applied Mathematics at the University of Würzburg in 1992. Since 1994 Christian Lubich is a full Professor of Numerical Mathematics at the [University of Tübingen](http://www.math.uni-tuebingen.de). Christian Lubich is a leading expert in numerical methods for evolution differential equations and a member of the editorial boards of top Numerical Analysis journals, like Mathematics of Computation, SIAM Journal on Numerical Analysis, SIAM Journal on Scientific Computing, Numerische Mathematik, Foundations of Computational Mathematics, IMA Journal of Numerical Analysis etc.

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**Πανεπιστήμιο Ιωαννίνων**